Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 09

B.Tech. (2011 onwards) (Sem.–1)

ENGINEERING MATHEMATICS-I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.
5. Use of non programmable calculator is allowed.

SECTION-A

l. Write briefly :

(a) Find the radius of curvature at the origin for the curve :

$y^4 + x^3 + a\,(x^2 + y^2) - a^2y = 0.$

(b) Find the area of the cardioad $r = a(1 - \cos\theta)$

(c) Find the volume of a sphere of radius 'a'.

(d) If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$.

(e) Find the equations of the tangent and normal to the surface $x^3 + y^3 + 3xyz = 3$ at (l,2,–l).

(f) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

(g) Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$.

(h) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$, where t is a time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.

(i) Find $div\,\vec{F}$ *and curl* $\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.

(j) State Stoke's theorem.

SECTION-B

2. Trace the folium of Descartes : $x^3 + y^3 = 3\,axy$ stating the salient points.

3. Find the moment of inertia of one loop of the lemniscates: $r^2 = a^2 \cos 2\theta$ about the initial line.

4. If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, then prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

5. Find the maximum and minimum distances of the point $A(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

SECTION-C

6. Evaluate (i) $\int_0^{\infty}\int_0^{x} xe^{-\frac{x^2}{y}}\,dy\,dx$ by change of order of integration,

(ii) $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}\left(x^2 + y^2 + z^2\right)dx\,dy\,dz.$

7. (i) Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point (2,–l, l).

(ii) A vector field is given by $\vec{F} = (\sin y)\,\hat{i} + x(1 + \cos y)\,\hat{j}$. Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$.

8. Apply Green's theorem to evaluate : $\int_C \left[y(1-\sin x)dx + \cos x\,dy\right]$, where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.

9. Verify divergence theorem for $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ taken over the curve bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.